Matlab Experiments

Convolution

Goal

We are going to write Matlab programs using convolution to perform calculations of sine wave and squares inputs applied to two low pass filters: an ideal low pass filter and an non-ideal filter using a simple RC circuit.

Impulse Response $h(t)$

- The impulse response to the Non-Ideal Low Pass (RC) filter is

$$h(t) = \frac{1}{RC} e^{-\frac{1}{RC}t} u(t)$$

- The impulse response to an Ideal Low Pass filter is

$$h(t) = 2 f_c \operatorname{sinc}(2\pi f_c t) \quad -\infty < t < \infty$$

where f_c is the cutoff frequency

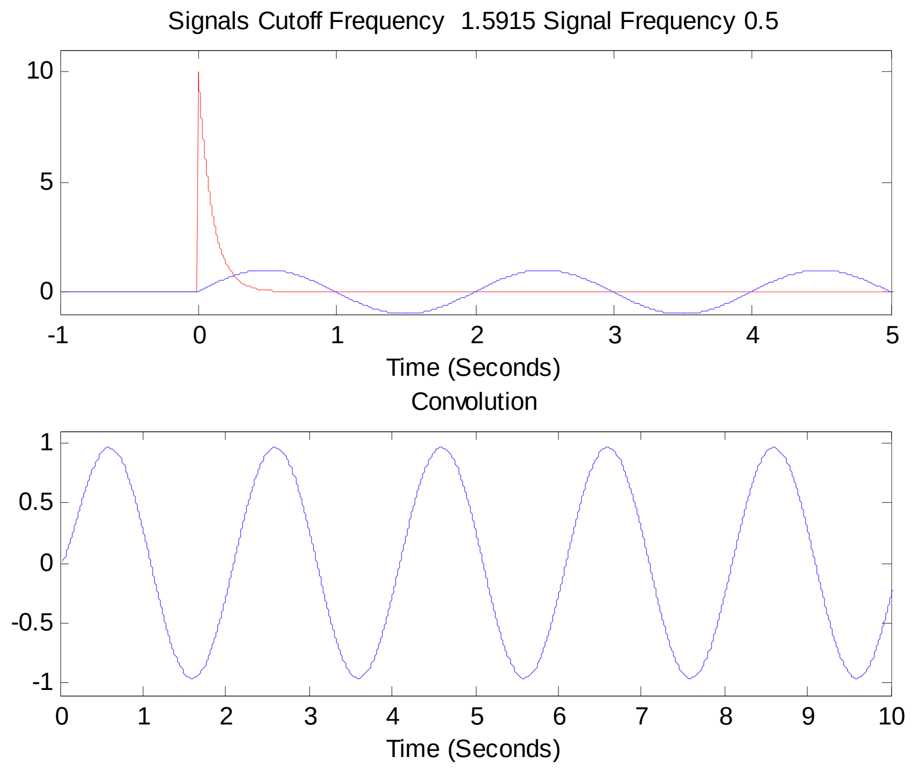
- See the Appendix for details

Experiments

1. Write a Matlab program to calculate the output for each of the types of filters: Ideal and RC.
2. The inputs will be sine and square waves.
3. The impulse responses are those given for each type of program.
4. The cutoff frequencies should be the same for each filter
5. The frequencies used for each of the input signals should be the same.
6. Perform at least (more would be preferable) 3 experiments for the sine wave input and at least (more would be preferable) 3 experiments for the square wave input for each of the two filters.
   a) Input frequency is less than the cutoff
   b) Input frequency is around the cutoff
   c) Several where the Input frequency is greater than the cutoff
   d) For the Ideal filter case with the square wave input find the frequency where the output is undistorted
7. The output of the program should plot the input signal, the impulse response, and the output signal. It should also display the cutoff frequency of the filter and the frequency of the input signal.
8. Write a professional report which includes the Matlab code used for the Ideal and RC filters, the Matlab outputs for each of the experiments, an explanation of the results, and comparison of the performance of these filters.

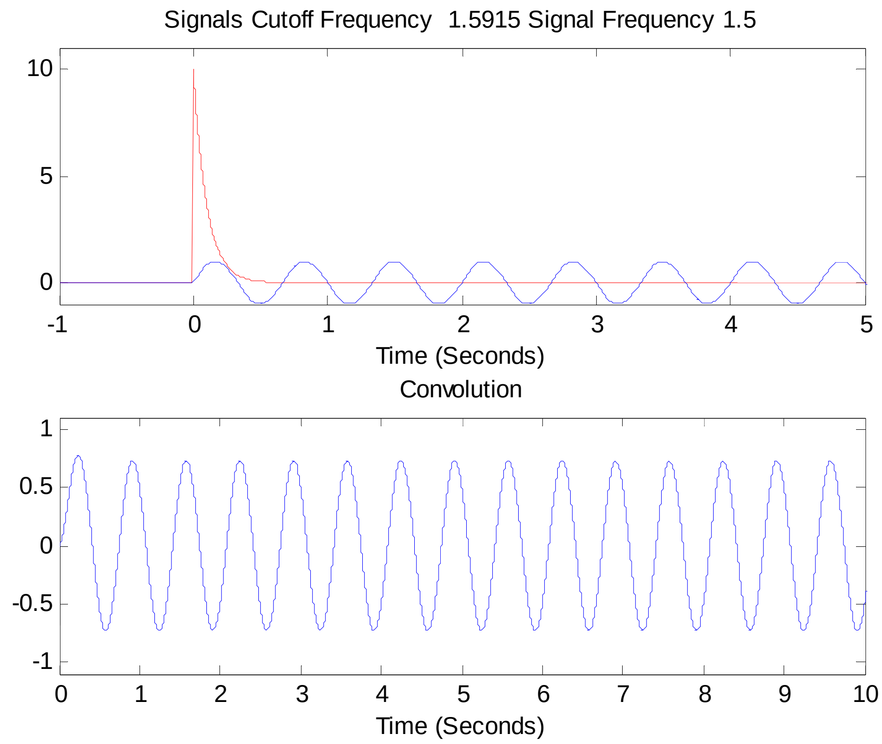
Examples

1. RC Filter
   A. Input: Sine Wave
      i. input frequency < cutoff frequency.



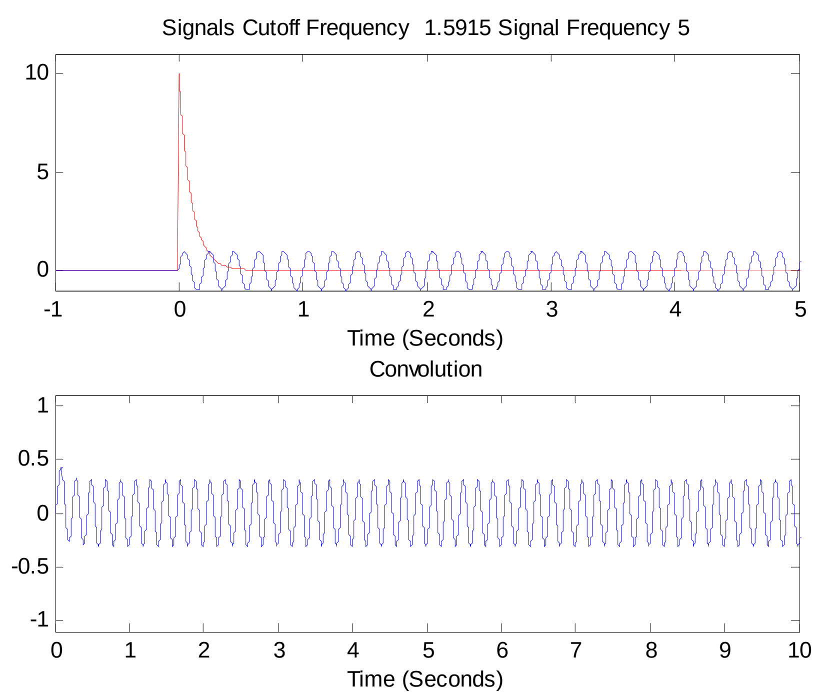
Examples

1. RC Filter
   A. Input: Sine Wave
      ii. input frequency close to the cutoff frequency.



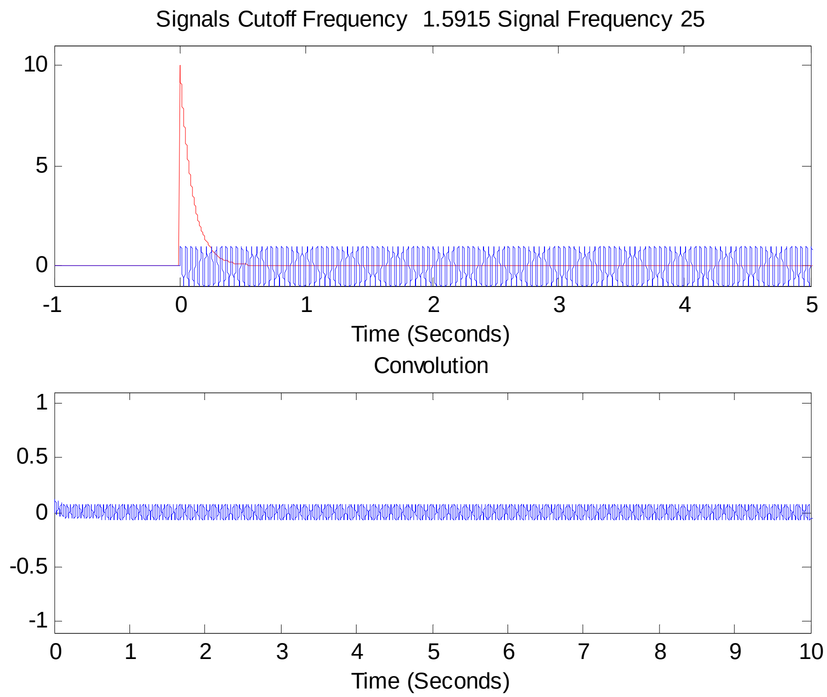
Examples

1. RC Filter
   A. Input: Sine Wave
      iii. input frequency > cutoff frequency.



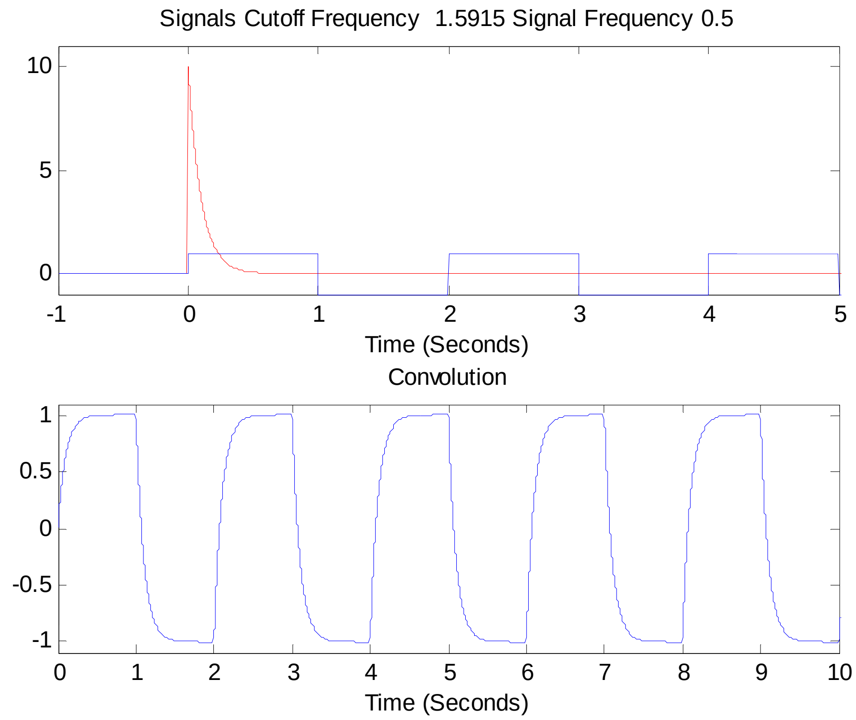
Examples

1. RC Filter
   A. Input: Sine Wave
      iv. input frequency >> cutoff frequency.



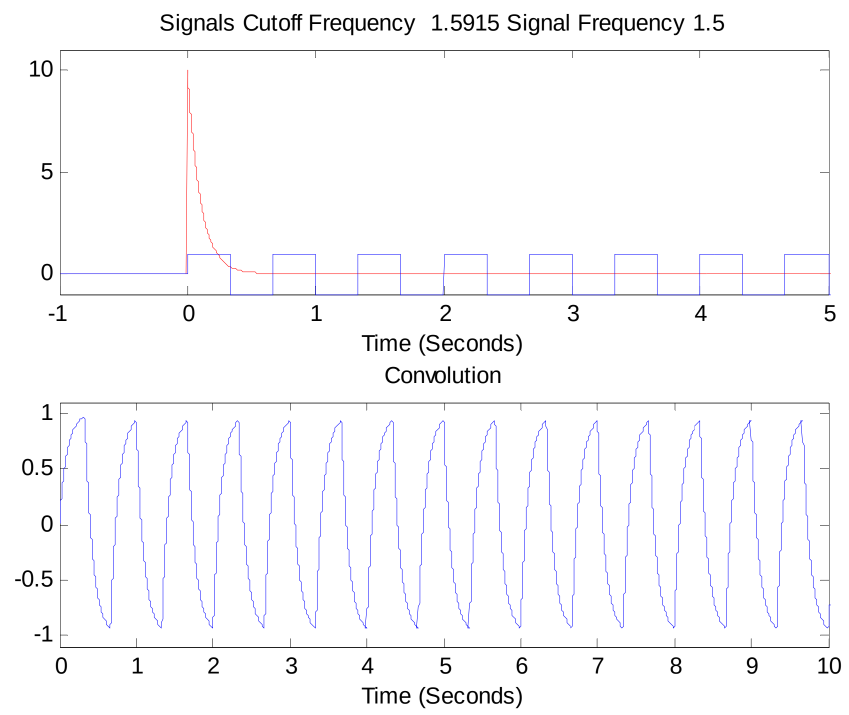
Examples

1. RC Filter
   B. Input: Square Wave
      i. input frequency < cutoff frequency.



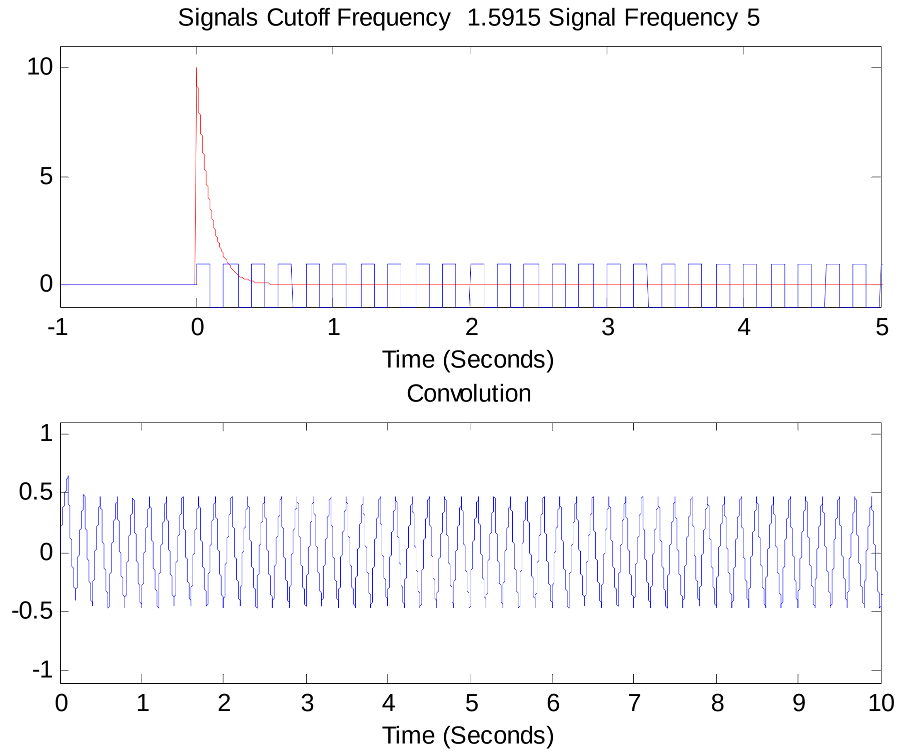
Examples

1. RC Filter
   B. Input: Square Wave
      ii. input frequency close to the cutoff frequency.



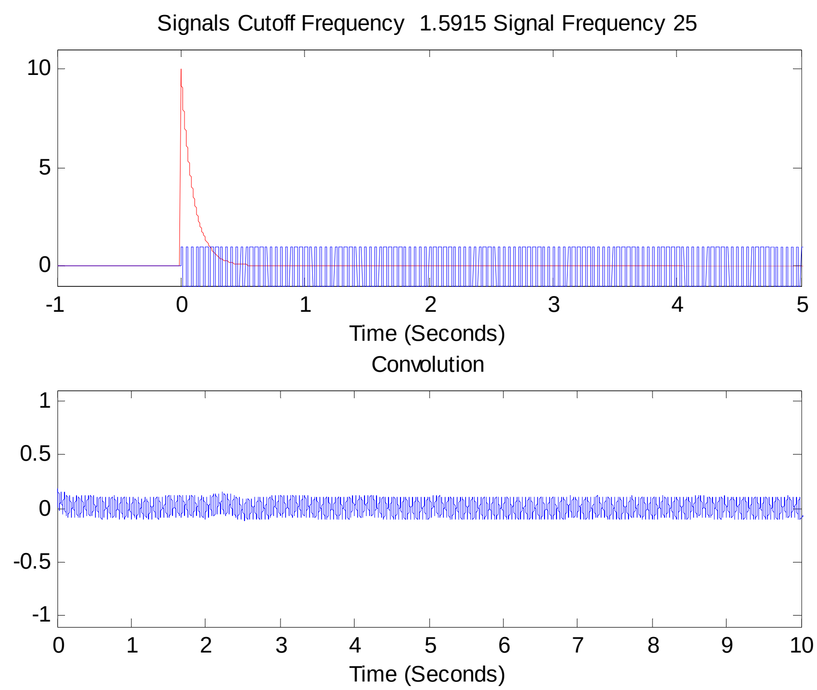
Examples

1. RC Filter
   B. Input: Sine Wave
      iii. input frequency > cutoff frequency.



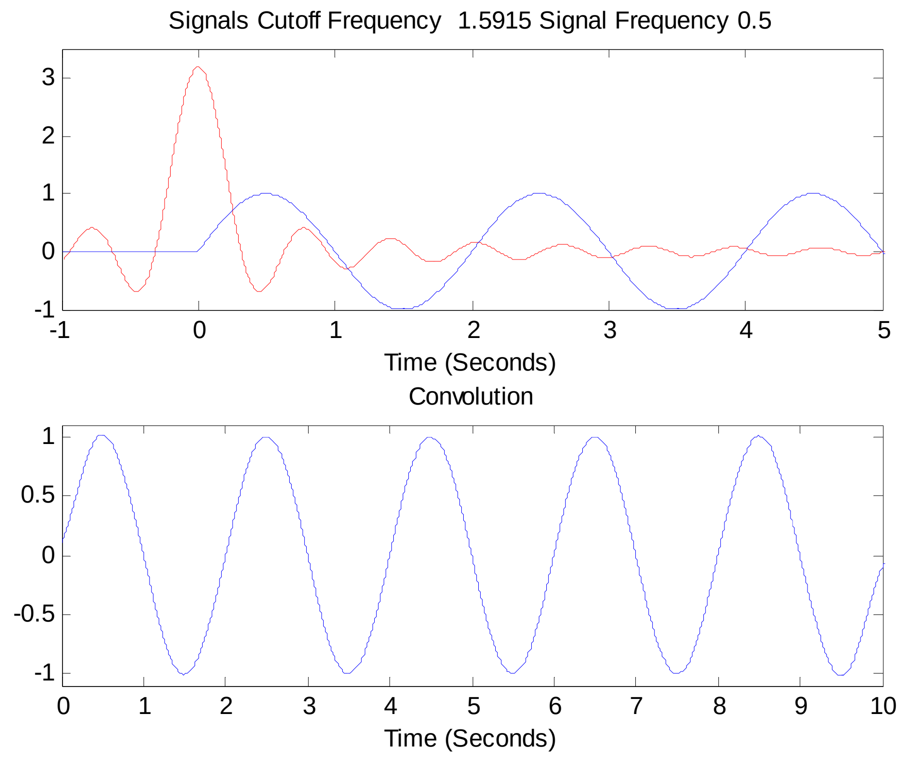
Examples

1. RC Filter
   B. Input: Square Wave
      iv. input frequency >> cutoff frequency.



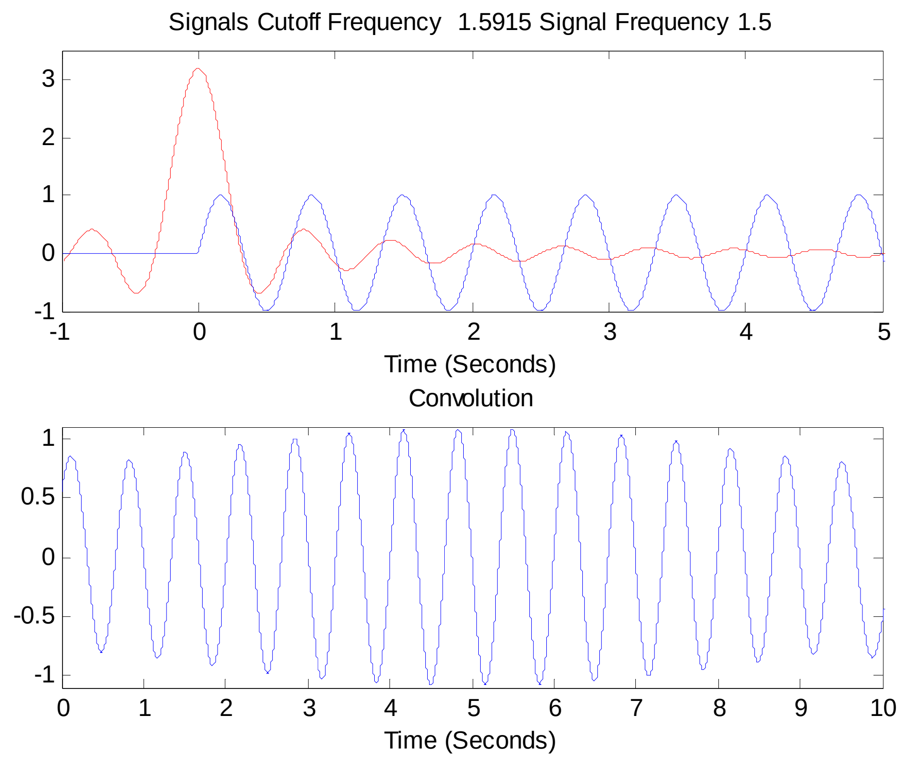
Examples

2. Ideal Filter
   A. Input: Sine Wave
      i. input frequency < cutoff frequency.



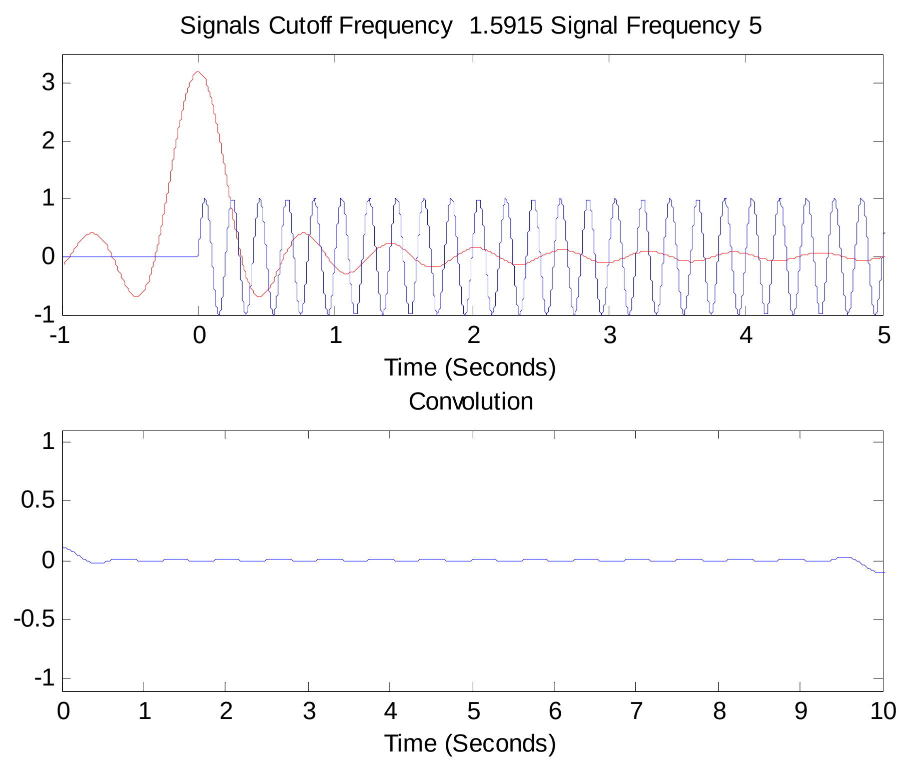
Examples

2. Ideal Filter
   A. Input: Sine Wave
      ii. input frequency close to the cutoff frequency.



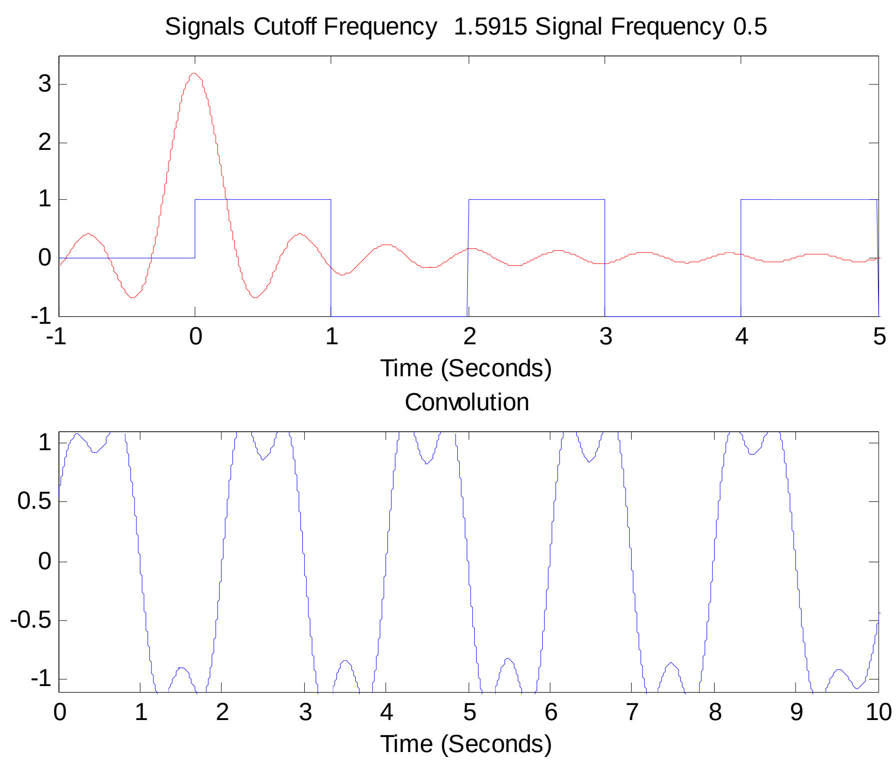
Examples

2. Ideal Filter
   A. Input: Sine Wave
      iii. input frequency > cutoff frequency.



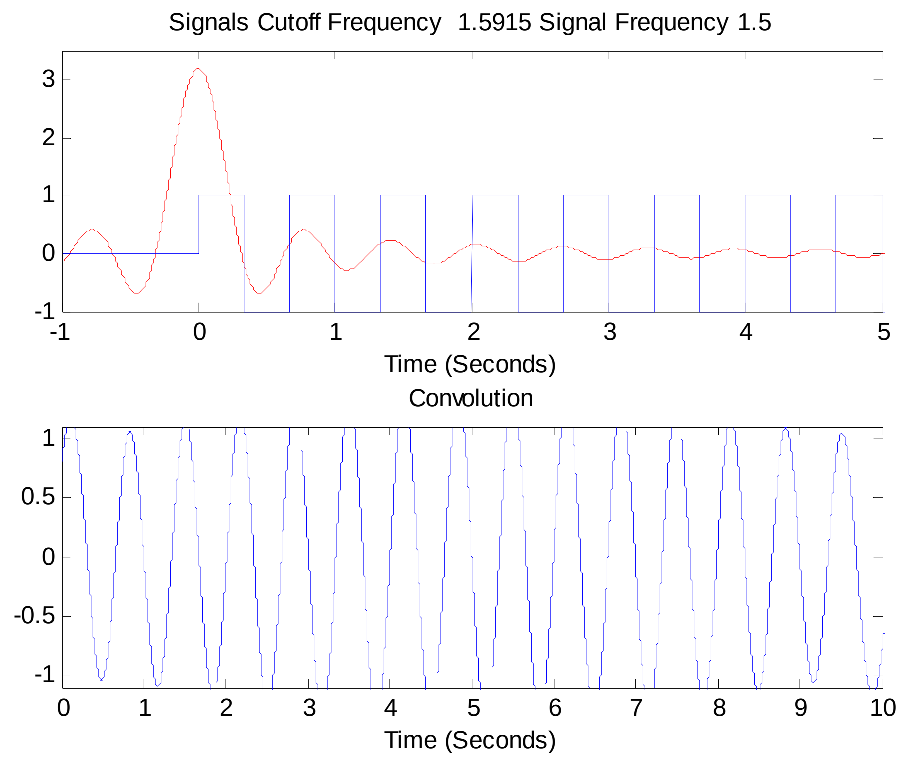
Examples

2. Ideal Filter
   B. Input: Square Wave
      i. input frequency < cutoff frequency.



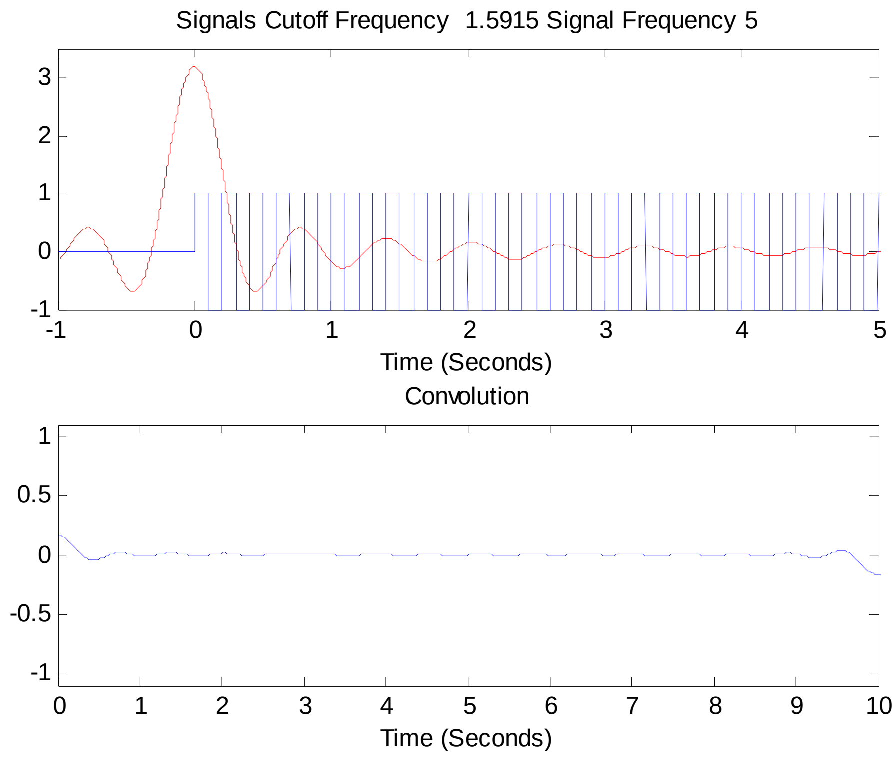
Examples

2. Ideal Filter
   B. Input: Square Wave
      ii. input frequency close to the cutoff frequency.



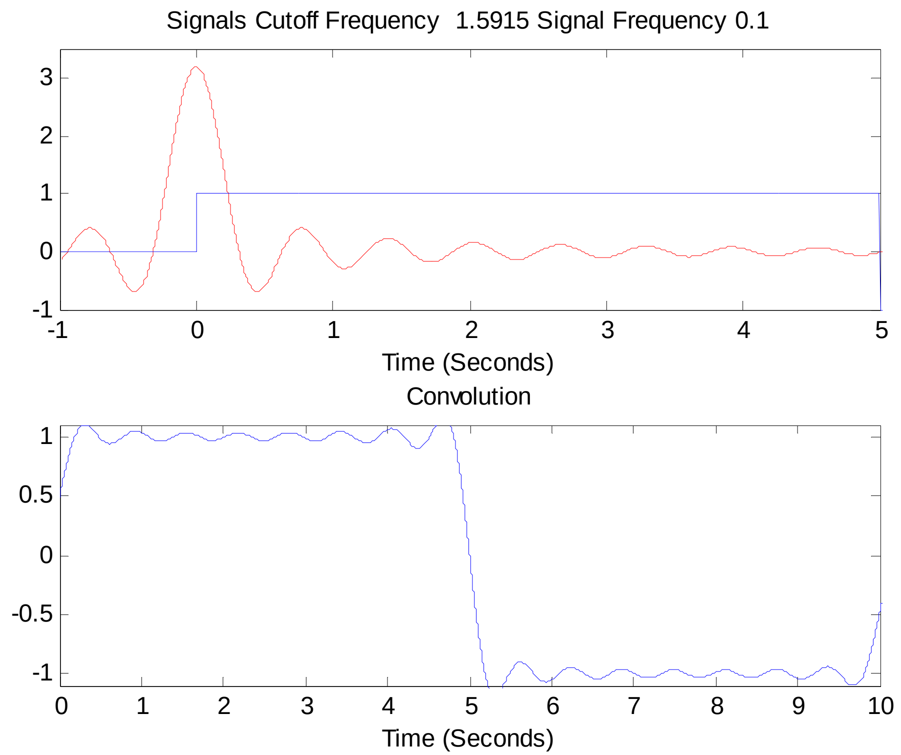
Examples

2. Ideal Filter
   B. Input: Sine Wave
      iii. input frequency > cutoff frequency.



Examples

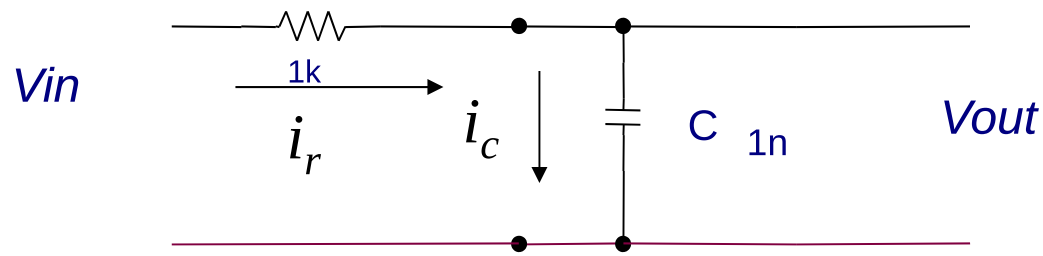
2. Ideal Filter
   B. Input: Square Wave
      iv. input frequency << cutoff frequency.



Some Hints

- Remember the "conv" is a discrete function and so the result must be multiplied by the sampling period.
- For the impulse response for the Ideal Filter
  1. Be careful on how you program its value for t=0.
  2. Recognize that $h(t)$ is valid for all values of t.

Appendix:
RC Low Pass Filter



$$i_c = i_r$$

$$i_c = C\frac{dVout}{dt} = i_r = \frac{Vin - Vout}{R}$$

$$C\frac{dVout}{dt} + \frac{Vout}{R} = \frac{Vin}{R}$$

$$(pC + \frac{1}{R})Vout = \frac{Vin}{R}$$

$$Vout = Ae^{-at} + B$$

$$pC + \frac{1}{R} = 0;\ p = \frac{1}{RC}$$

$$B = Vin$$

$$Vout = Ae^{-\frac{1}{RC}t} + Vin$$

$$Vout(0) = 0 = Ae^{-\frac{1}{RC}t} + Vin$$

$$A = -Vin$$

$$Vout = Vin(1 - e^{-\frac{1}{RC}t})u(t)$$

$$h(t) = \frac{d(1-e^{-\frac{1}{RC}t})u(t)}{dt} = (1-e^{-\frac{1}{RC}t})\delta(t) - ((-\frac{1}{RC})e^{-\frac{1}{RC}t})u(t) = \frac{1}{RC}e^{-\frac{1}{RC}t}u(t)$$

$$Vout = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}Vin$$

$$\left|\frac{Vout}{Vin}\right| = \left|\frac{1}{1 + j\omega CR}\right| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

The cutoff (radian) frequency is defined as $\omega_c = \frac{1}{RC}$.

Appendix
Ideal Low Pass Filter

It can be shown that the impulse response to an ideal low pass filter is

$$h(t) = 2f_c \operatorname{sinc}(2\pi f_c t) \quad -\infty < t < \infty$$

where f_c is the cutoff frequency and the "sinc" function is given as:

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$$